Grade 11/12 Math Circles

March 20, 2024

Primality Testing and Integer Factorization - Problem Set

1. Determine whether 161 is prime, and if not, factor it.
2. Calculate the prime factorization of 1001.
3. Prove that if a divides n and $\sqrt{n} \leq a < n$, then there exists b which divides n and satisfies $1 < b \leq \sqrt{n}$.
4. Determine whether 1739 and 1741 are prime, and if not, factor them.
5. Find the prime factorization of 344929.
6. Using the prime number theorem, approximately how many primes are less than 100?
7. Suggest an algorithm to calculate the primes between two positive real numbers x and y (for example, $x = 100$ and $y = 100$). Notice that the Sieve of Eratosthenes would not work without modification, since 2 would never be detected as a prime and thus even numbers would not be struck out.
8. Find a factor of 999991.
9. (Challenge) Find a factor of 2146681.